Volume Using Cross Sections

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Topic tags have been given for each question to enable you to know if you can do the question or whether you need to wait to cover the additional topic(s).

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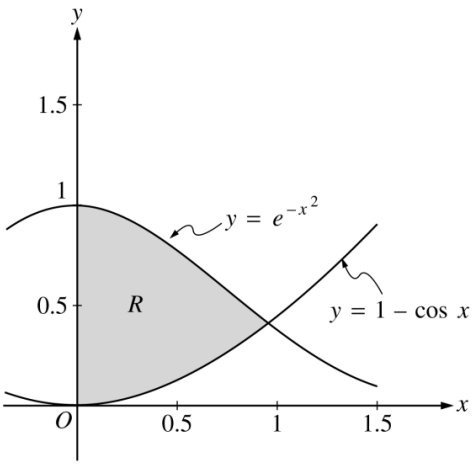
Question 1

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Volume using Cross Sections, Integration - Area Between Curves, Volume of Revolution – Washer Method, Integration Technique – Trigonometry, Integration Technique – Exponentials

Paper: Part A-Calc / Series: 2000 / Difficulty: Easy / Question Number: 1



1. Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.

   (a) Find the area of the region R.

   (b) Find the volume of the solid generated when the region R is revolved about the x-axis.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

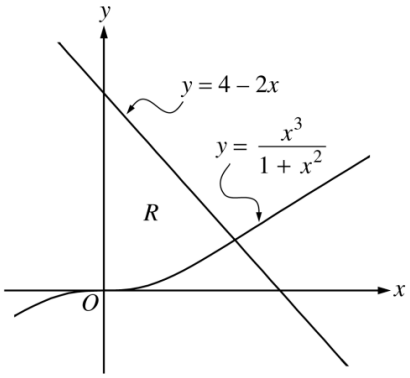
Question 2

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method, Volume using Cross Sections

Paper: Part A-Calc / Series: 2002-Form-B / Difficulty: Easy / Question Number: 1



1. Let R be the region bounded by the y-axis and the graphs of $y = \frac{x^3}{1 + x^2}$ and $y = 4 - 2x$, as shown in the figure above.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the x-axis.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

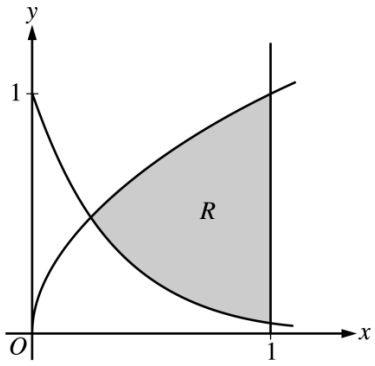
Question 3

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method, Volume using Cross Sections

Paper: Part A-Calc / Series: 2003 / Difficulty: Easy / Question Number: 1



1. Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.

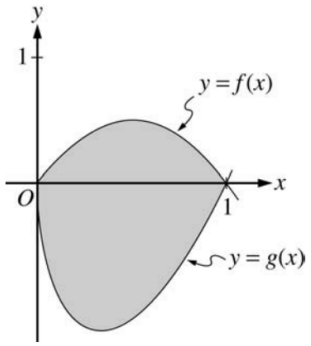
Question 4

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method, Volume using Cross Sections

Paper: Part A-Calc / Series: 2004 / Difficulty: Somewhat Challenging / Question Number: 2



2. Let f and g be the functions given by $f(x) = 2x(1 - x)$ and $g(x) = 3(x - 1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and g are shown in the figure above.

   (a) Find the area of the shaded region enclosed by the graphs of f and g.

   (b) Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$.

   (c) Let h be the function given by $h(x) = kx(1 - x)$ for $0 \le x \le 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k.

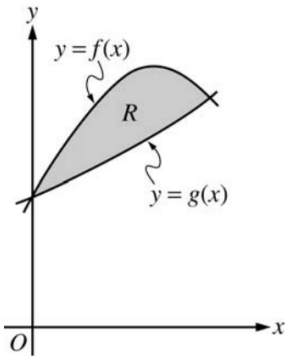
Question 5

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Volume using Cross Sections, Integration - Area Between Curves, Volume of Revolution – Washer Method

Paper: Part A-Calc / Series: 2005-Form-B / Difficulty: Medium / Question Number: 1



1. Let f and g be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let R be the shaded region in the first quadrant enclosed by the graphs of f and g as shown in the figure above.
   (a) Find the area of R.
   (b) Find the volume of the solid generated when R is revolved about the x-axis.
   (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.

Question 6

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Under A Curve, Volume of Revolution – Washer Method, Volume using Cross Sections

Paper: Part A-Calc / Series: 2007 / Difficulty: Somewhat Challenging / Question Number: 1

1. Let R be the region in the first and second quadrants bounded above by the graph of $y = \frac{20}{1+x^2}$ and below by the horizontal line $y = 2$.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is rotated about the x-axis.

   (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume of this solid.

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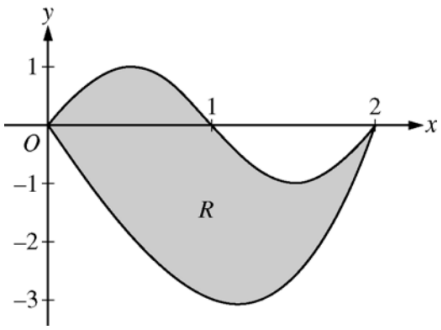
Question 7

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume using Cross Sections

Paper: Part A-Calc / Series: 2008 / Difficulty: Medium / Question Number: 1



1. Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.

   (a) Find the area of R.

   (b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

   (d) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

Question 8

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method, Volume using Cross Sections

Paper: Part A-Calc / Series: 2008-Form-B / Difficulty: Easy / Question Number: 1

1. Let R be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is rotated about the vertical line $x = -1$.

   (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the y-axis are squares. Find the volume of this solid.

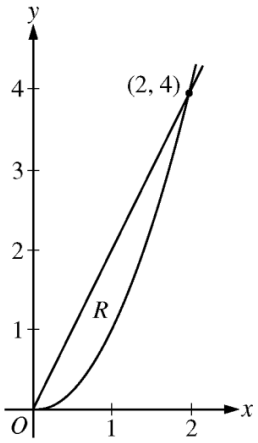
Question 9

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume using Cross Sections, Integration Technique – Trigonometry, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2009 / Difficulty: Easy / Question Number: 4



4. Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.

   (a) Find the area of R.

   (b) The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.

   (c) Another solid has the same base R. For this solid, the cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

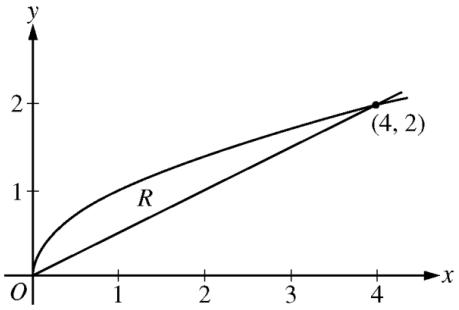
Question 10

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume using Cross Sections, Volume of Revolution – Washer Method, Integration Technique - Harder Powers

Paper: Part B-Non-Calc / Series: 2009-Form-B / Difficulty: Easy / Question Number: 4



4. Let R be the region bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{2}$, as shown in the figure above.

   (a) Find the area of R.

   (b) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are squares. Find the volume of this solid.

   (c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 2$.

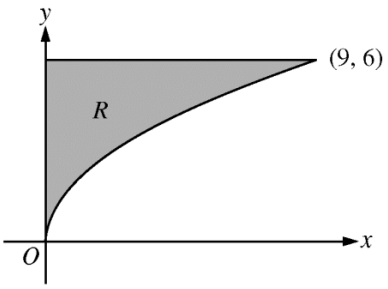
Question 11

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method, Volume using Cross Sections

Paper: Part B-Non-Calc / Series: 2010 / Difficulty: Easy / Question Number: 4



4. Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.

   (a) Find the area of R.

   (b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

   (c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

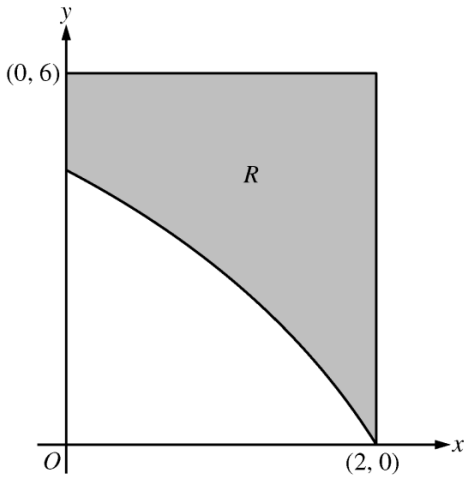
Question 12

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method, Volume using Cross Sections

Paper: Part A-Calc / Series: 2010-Form-B / Difficulty: Easy / Question Number: 1



1. In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.
